Zadatak 141 (Ajax, maturant)

Odredite y iz sustava jednadžba $\begin{cases} 3\cdot x+8\cdot y+12=0 \\ 4\cdot y^2=9\cdot x \end{cases}.$

Rješenje 141

Ponovimo!

$$a^2+2\cdot a\cdot b+b^2=(a+b)^2 \quad , \quad a^2=0 \Rightarrow a=0.$$

$$\left.\begin{array}{l} 3\cdot x+8\cdot y+12=0 \\ 4\cdot y^2=9\cdot x \end{array}\right\} \Rightarrow \left.\begin{array}{l} 3\cdot x+8\cdot y+12=0 \;/\cdot 3 \\ 4\cdot y^2=9\cdot x \end{array}\right\} \Rightarrow \left.\begin{array}{l} 9\cdot x+24\cdot y+36=0 \\ 4\cdot y^2=9\cdot x \end{array}\right\} \Rightarrow$$

$$\Rightarrow \begin{bmatrix} \text{metoda} \\ \text{zamjene} \end{bmatrix} \Rightarrow 4\cdot y^2+24\cdot y+36=0 \Rightarrow 4\cdot y^2+24\cdot y+36=0 \;/:4 \Rightarrow$$

$$\Rightarrow y^2+6\cdot y+9=0 \Rightarrow (y+3)^2=0 \Rightarrow y+3=0 \Rightarrow y=-3.$$

Vježba 141

Odredite x iz sustava jednadžba $\begin{cases} 3\cdot x+8\cdot y+12=0 \\ 4\cdot y^2=9\cdot x \end{cases}.$

Rezultat: x = 4.

Zadatak 142 (Dora1, veleučilište)

Riješite sustav jednadžba:

$$\begin{cases} \quad\;\; x_2+x_3+\ldots+x_{n-1}+x_n=1 \\ x_1 \quad\;\; +x_3+\ldots+x_{n-1}+x_n=2 \\ x_1+x_2 \quad\;\; +\ldots+x_{n-1}+x_n=3 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ x_1+x_2+x_3+\ldots+x_{n-1} \quad\;\; =n \end{cases} , \quad x_i\in R \;,\; i=1, 2, 3, \ldots, n \;,\; n\in N.$$

Rješenje 142

Ponovimo!

$$a=b \text{ i } c=d \Rightarrow a+c=b+d \quad , \quad n=\frac{n}{1} \quad , \quad \frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d} \quad , \quad 1+2+3+\ldots+n=\frac{n\cdot(n+1)}{2}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Uvedemo zamjenu

$$x_1+x_2+x_3+\ldots+x_{n-1}+x_n=t$$

pa dobivamo sustav jednadžba:

$$\left.\begin{array}{l} \quad\;\; x_2+x_3+\ldots+x_{n-1}+x_n=1 \\ x_1 \quad\;\; +x_3+\ldots+x_{n-1}+x_n=2 \\ x_1+x_2 \quad\;\; +\ldots+x_{n-1}+x_n=3 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ x_1+x_2+x_3+\ldots+x_{n-1} \quad\;\; =n \end{array}\right\} \Rightarrow \begin{bmatrix} \text{zamjena} \\ x_1+x_2+x_3+\ldots+x_{n-1}+x_n=t \end{bmatrix} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} t-x_1=1 \\ t-x_2=2 \\ t-x_3=3 \\ \dots\dots\dots\dots \\ t-x_n=n \end{array}\right\} \Rightarrow \boxed{\begin{array}{l}\text{može i kraći zapis} \\ t-x_i=i, \\ i=1,\ 2,\ 3,\ \dots,\ n\end{array}} \Rightarrow \left[\begin{array}{c}\text{zbrojimo} \\ \text{jednadžbe}\end{array}\right] \Rightarrow$$

$$\Rightarrow n\cdot t-x_1-x_2-x_3-\dots-x_n=1+2+3+\dots+n \Rightarrow$$

$$\Rightarrow n\cdot t-\left(x_1+x_2+x_3+\dots+x_n\right)=\frac{n\cdot(n+1)}{2} \Rightarrow n\cdot t-t=\frac{n\cdot(n+1)}{2} \Rightarrow$$

$$\Rightarrow (n-1)\cdot t=\frac{n\cdot(n+1)}{2} \Rightarrow (n-1)\cdot t=\frac{n\cdot(n+1)}{2} \ /\cdot\frac{1}{n-1} \Rightarrow t=\frac{n\cdot(n+1)}{2\cdot(n-1)}.$$

Vraćamo se zamjeni.

$$\left.\begin{array}{l} t-x_1=1 \\ t-x_2=2 \\ t-x_3=3 \\ \dots\dots\dots \\ t-x_n=n \end{array}\right\} \Rightarrow \left.\begin{array}{l} -x_1=1-t \\ -x_2=2-t \\ -x_3=3-t \\ \dots\dots\dots \\ -x_n=n-t \end{array}\right\} \Rightarrow \left.\begin{array}{l} -x_1=1-t \ /\cdot(-1) \\ -x_2=2-t \ /\cdot(-1) \\ -x_3=3-t \ /\cdot(-1) \\ \dots\dots\dots\dots\dots \\ -x_n=n-t \ /\cdot(-1) \end{array}\right\} \Rightarrow \left.\begin{array}{l} x_1=t-1 \\ x_2=t-2 \\ x_3=t-3 \\ \dots\dots\dots \\ x_n=t-n \end{array}\right\} \Rightarrow$$

$$\Rightarrow \boxed{\begin{array}{l}\text{može i kraći zapis} \\ x_i=t-i, \\ i=1,\ 2,\ 3,\ \dots,\ n\end{array}} \Rightarrow \left[\begin{array}{c}\text{zamjena} \\ t=\frac{n\cdot(n+1)}{2\cdot(n-1)}\end{array}\right] \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} x_1=\frac{n\cdot(n+1)}{2\cdot(n-1)}-1 \\ x_2=\frac{n\cdot(n+1)}{2\cdot(n-1)}-2 \\ x_3=\frac{n\cdot(n+1)}{2\cdot(n-1)}-3 \\ \dots\dots\dots\dots\dots \\ x_n=\frac{n\cdot(n+1)}{2\cdot(n-1)}-n \end{array}\right\} \Rightarrow \boxed{\begin{array}{l}\text{može i kraći zapis} \\ x_i=\frac{n\cdot(n+1)}{2\cdot(n-1)}-i, \\ i=1,\ 2,\ 3,\ \dots,\ n\end{array}} \Rightarrow$$

$$\Rightarrow \left.\begin{aligned} x_1 &= \frac{n\cdot(n+1)}{2\cdot(n-1)}-\frac{1}{1} \\ x_2 &= \frac{n\cdot(n+1)}{2\cdot(n-1)}-\frac{2}{1} \\ x_3 &= \frac{n\cdot(n+1)}{2\cdot(n-1)}-\frac{3}{1} \\ &\dots\dots\dots \\ x_n &= \frac{n\cdot(n+1)}{2\cdot(n-1)}-\frac{n}{1} \end{aligned}\right\} \Rightarrow \left.\begin{aligned} x_1 &= \frac{n\cdot(n+1)-2\cdot(n-1)}{2\cdot(n-1)} \\ x_2 &= \frac{n\cdot(n+1)-4\cdot(n-1)}{2\cdot(n-1)} \\ x_3 &= \frac{n\cdot(n+1)-6\cdot(n-1)}{2\cdot(n-1)} \\ &\dots\dots\dots \\ x_n &= \frac{n\cdot(n+1)-2\cdot n\cdot(n-1)}{2\cdot(n-1)} \end{aligned}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{aligned} x_1 &= \frac{n^2+n-2\cdot n+2}{2\cdot(n-1)} \\ x_2 &= \frac{n^2+n-4\cdot n+4}{2\cdot(n-1)} \\ x_3 &= \frac{n^2+n-6\cdot n+6}{2\cdot(n-1)} \\ &\dots\dots\dots \\ x_n &= \frac{n^2+n-2\cdot n^2+2\cdot n}{2\cdot(n-1)} \end{aligned}\right\} \Rightarrow \left.\begin{aligned} x_1 &= \frac{n^2-n+2}{2\cdot(n-1)} \\ x_2 &= \frac{n^2-3\cdot n+4}{2\cdot(n-1)} \\ x_3 &= \frac{n^2-5\cdot n+6}{2\cdot(n-1)} \\ &\dots\dots\dots \\ x_n &= \frac{3\cdot n-n^2}{2\cdot(n-1)} \end{aligned}\right\}.$$

Vježba 142

Odmor!

Rezultat: …

Zadatak 143 (Lony, gimnazija)

Riješite sustav jednadžba:

$$|x-y|=2\ ,\ |x|+|y|=4\ ,\ x,\ y\in R.$$

Rješenje 143

Ponovimo!

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad ,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Za realni broj x njegova je apsolutna vrijednost (modul) broj $|x|$ koji određujemo na ovaj način:

$$|x|=\begin{cases} x, & x\geq 0 \\ -x, & x<0. \end{cases}$$

Ako je broj x pozitivan ili nula, tada je on jednak svojoj apsolutnoj vrijednosti. Za svaki x, $x \geq 0$, vrijedi $|x| = x$.

Ako je x negativan broj, njegova apsolutna vrijednost je suprotan broj – x koji je pozitivan. Za svaki x, $x < 0$, je $|x| = -x$.

Postoje četiri slučaja!

Prvi slučaj
$x \geq 0\ ,\ y \geq 0$

♥ $$x \geq y \geq 0 \Rightarrow \left.\begin{matrix} x-y \geq 0 \\ x \geq 0 \\ y \geq 0 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} |x-y| = x-y \\ |x| = x \\ |y| = y \end{matrix}\right\} \Rightarrow \left[\begin{matrix} \text{sustav} \\ |x-y| = 2 \\ |x|+|y| = 4 \end{matrix}\right] \Rightarrow \left.\begin{matrix} x-y=2 \\ x+y=4 \end{matrix}\right\} \Rightarrow$$

$$\Rightarrow \left[\begin{matrix} \text{metoda suprotnih} \\ \text{koeficijenata} \end{matrix}\right] \Rightarrow 2 \cdot x = 6 \Rightarrow 2 \cdot x = 6\ /:2 \Rightarrow x = 3.$$

Računamo y.

$$\left.\begin{matrix} x=3 \\ x+y=4 \end{matrix}\right\} \Rightarrow 3+y=4 \Rightarrow y=4-3 \Rightarrow y=1.$$

Rješenje je

$$(x_1, y_1) = (3, 1).$$

♥♥ $$y \geq x \geq 0 \Rightarrow \left.\begin{matrix} x-y \leq 0 \\ x \geq 0 \\ y \geq 0 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} |x-y| = -(x-y) \\ |x| = x \\ |y| = y \end{matrix}\right\} \Rightarrow \left.\begin{matrix} |x-y| = -x+y \\ |x| = x \\ |y| = y \end{matrix}\right\} \Rightarrow$$

$$\Rightarrow \left[\begin{matrix} \text{sustav} \\ |x-y| = 2 \\ |x|+|y| = 4 \end{matrix}\right] \Rightarrow \left.\begin{matrix} -x+y=2 \\ x+y=4 \end{matrix}\right\} \Rightarrow \left[\begin{matrix} \text{metoda suprotnih} \\ \text{koeficijenata} \end{matrix}\right] \Rightarrow 2 \cdot y = 6 \Rightarrow 2 \cdot y = 6\ /:2 \Rightarrow y = 3.$$

Računamo x.

$$\left.\begin{matrix} y=3 \\ x+y=4 \end{matrix}\right\} \Rightarrow x+3=4 \Rightarrow x=4-3 \Rightarrow x=1.$$

Rješenje je

$$(x_2, y_2) = (1, 3).$$

Drugi slučaj
$x < 0\ ,\ y \geq 0$

$$x < 0\ ,\ y \geq 0 \Rightarrow \left.\begin{matrix} x-y < 0 \\ x < 0 \\ y \geq 0 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} |x-y| = -(x-y) \\ |x| = -x \\ |y| = y \end{matrix}\right\} \Rightarrow \left.\begin{matrix} |x-y| = -x+y \\ |x| = -x \\ |y| = y \end{matrix}\right\} \Rightarrow$$

$$\Rightarrow \left[\begin{matrix} \text{sustav} \\ |x-y| = 2 \\ |x|+|y| = 4 \end{matrix}\right] \Rightarrow \left.\begin{matrix} x-y=2 \\ x+y=4 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} -x+y=2 \\ -x+y=4 \end{matrix}\right\} \Rightarrow \left[\begin{matrix} \text{kontradikcija} \\ \text{suprotnost} \\ \text{proturječje} \\ \text{protuslovnost} \end{matrix}\right] \Rightarrow \text{nema rješenja}.$$

Treći slučaj
$x < 0\ ,\ y < 0$

♥ $$y \leq x < 0 \Rightarrow \left.\begin{matrix} x-y \geq 0 \\ x < 0 \\ y < 0 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} |x-y| = x-y \\ |x| = -x \\ |y| = -y \end{matrix}\right\} \Rightarrow \left[\begin{matrix} \text{sustav} \\ |x-y| = 2 \\ |x|+|y| = 4 \end{matrix}\right] \Rightarrow \left.\begin{matrix} x-y=2 \\ -x-y=4 \end{matrix}\right\} \Rightarrow$$

$$\Rightarrow \begin{bmatrix} \text{metoda suprotnih} \\ \text{koeficijenata} \end{bmatrix} \Rightarrow -2\cdot y=6 \Rightarrow -2\cdot y=6 \ /:(-2) \Rightarrow y=-3.$$

Računamo x.

$$\left.\begin{matrix} y=-3 \\ x-y=2 \end{matrix}\right\} \Rightarrow x-(-3)=2 \Rightarrow x+3=2 \Rightarrow x=2-3 \Rightarrow x=-1.$$

Rješenje je

$$(x_3, y_3)=(-1, -3).$$

♥♥ $$x\le y<0 \Rightarrow \left.\begin{matrix} x-y<0 \\ x<0 \\ y<0 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} |x-y|=-(x-y) \\ |x|=-x \\ |y|=-y \end{matrix}\right\} \Rightarrow \left.\begin{matrix} |x-y|=-x+y \\ |x|=-x \\ |y|=-y \end{matrix}\right\} \Rightarrow$$

$$\Rightarrow \begin{bmatrix} \text{sustav} \\ |x-y|=2 \\ |x|+|y|=4 \end{bmatrix} \Rightarrow \left.\begin{matrix} -x+y=2 \\ -x-y=4 \end{matrix}\right\} \Rightarrow \begin{bmatrix} \text{metoda suprotnih} \\ \text{koeficijenata} \end{bmatrix} \Rightarrow -2\cdot x=6 \Rightarrow -2\cdot x=6 \ /:(-2) \Rightarrow x=-3.$$

Računamo y.

$$\left.\begin{matrix} x=-3 \\ -x+y=2 \end{matrix}\right\} \Rightarrow -(-3)+y=2 \Rightarrow 3+y=2 \Rightarrow y=2-3 \Rightarrow y=-1.$$

Rješenje je

$$(x_4, y_4)=(-3, -1).$$

Četvrti slučaj
$x\ge 0\ ,\ y<0$

$$x\ge 0\ ,\ y<0 \Rightarrow \left.\begin{matrix} x-y\ge 0 \\ x\ge 0 \\ y<0 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} |x-y|=x-y \\ |x|=x \\ |y|=-y \end{matrix}\right\} \Rightarrow$$

$$\Rightarrow \begin{bmatrix} \text{sustav} \\ |x-y|=2 \\ |x|+|y|=4 \end{bmatrix} \Rightarrow \left.\begin{matrix} x-y=2 \\ x-y=4 \end{matrix}\right\} \Rightarrow \begin{bmatrix} \text{kontradikcija} \\ \text{suprotnost} \\ \text{proturječje} \\ \text{protuslovnost} \end{bmatrix} \Rightarrow \text{nema rješenja.}$$

Vježba 143

Riješite sustav jednadžba:

$$|x-1|+|y-5|=1\ ,\ |x-1|-y+5=0.$$

Rezultat: $\left(\frac{1}{2}, \frac{11}{2}\right)\ ,\ \left(\frac{3}{2}, \frac{11}{2}\right).$

Zadatak 144 (Zekoslav, gimnazija)

Riješite sustav jednadžba:

$$x-y+z-v=\frac{1}{2}\quad ,\quad x+y-z-v=\frac{1}{3}\quad ,\quad x-y-z+v=\frac{1}{6}\quad ,\quad x+y+z+v=7.$$

Rješenje 144

Ponovimo!

$$n=\frac{n}{1} \quad , \quad \frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d} \quad , \quad \frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d} \quad , \quad a=b \text{ i } c=d \Rightarrow a+c=b+d.$$

$$\frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \quad , \quad n\neq 0 \quad , \quad n\neq 1.$$

Zbrojimo sve četiri jednadžbe.

$$\left.\begin{array}{l} x-y+z-v=\frac{1}{2} \\ x+y-z-v=\frac{1}{3} \\ x-y-z+v=\frac{1}{6} \\ x+y+z+v=7 \end{array}\right\} \Rightarrow \left[\begin{array}{l}\text{zbrojimo} \\ \text{jednadžbe}\end{array}\right] \Rightarrow$$

$$\Rightarrow x-y+z-v+x+y-z-v+x-y-z+v+x+y+z+v=\frac{1}{2}+\frac{1}{3}+\frac{1}{6}+7 \Rightarrow$$

$$\Rightarrow x-y+z-v+x+y-z-v+x-y-z+v+x+y+z+v=\frac{1}{2}+\frac{1}{3}+\frac{1}{6}+\frac{7}{1} \Rightarrow$$

$$\Rightarrow x+x+x+x=\frac{3+2+1+42}{6} \Rightarrow 4\cdot x=\frac{48}{6} \Rightarrow 4\cdot x=\frac{48}{6} \Rightarrow 4\cdot x=8 \Rightarrow$$

$$\Rightarrow 4\cdot x=8 \; /:4 \Rightarrow x=2.$$

Zatim zbrojimo treću i četvrtu jednadžbu.

$$\left.\begin{array}{l} x-y-z+v=\frac{1}{6} \\ x+y+z+v=7 \end{array}\right\} \Rightarrow \left[\begin{array}{l}\text{zbrojimo} \\ \text{jednadžbe}\end{array}\right] \Rightarrow x-y-z+v+x+y+z+v=\frac{1}{6}+7 \Rightarrow$$

$$\Rightarrow x-y-z+v+x+y+z+v=\frac{1}{6}+\frac{7}{1} \Rightarrow x+v+x+v=\frac{1+42}{6} \Rightarrow 2\cdot x+2\cdot v=\frac{43}{6} \Rightarrow$$

$$\Rightarrow \left[x=2\right] \Rightarrow 2\cdot 2+2\cdot v=\frac{43}{6} \Rightarrow 4+2\cdot v=\frac{43}{6} \Rightarrow 2\cdot v=\frac{43}{6}-4 \Rightarrow 2\cdot v=\frac{43}{6}-\frac{4}{1} \Rightarrow$$

$$\Rightarrow 2\cdot v=\frac{43-24}{6} \Rightarrow 2\cdot v=\frac{19}{6} \Rightarrow 2\cdot v=\frac{19}{6} \; /\cdot\frac{1}{2} \Rightarrow v=\frac{19}{12}.$$

Zbrojimo drugu i treću jednadžbu.

$$\left.\begin{array}{l} x+y-z-v=\frac{1}{3} \\ x-y-z+v=\frac{1}{6} \end{array}\right\} \Rightarrow \left[\begin{array}{l}\text{zbrojimo} \\ \text{jednadžbe}\end{array}\right] \Rightarrow x+y-z-v+x-y-z+v=\frac{1}{3}+\frac{1}{6} \Rightarrow$$

$$\Rightarrow x+y-z-v+x-y-z+v=\frac{2+1}{6} \Rightarrow x-z+x-z=\frac{3}{6} \Rightarrow 2\cdot x-2\cdot z=\frac{3}{6} \Rightarrow$$

$$\Rightarrow 2\cdot x-2\cdot z=\frac{1}{2} \Rightarrow \left[x=2\right] \Rightarrow 2\cdot 2-2\cdot z=\frac{1}{2} \Rightarrow 4-2\cdot z=\frac{1}{2} \Rightarrow -2\cdot z=\frac{1}{2}-4 \Rightarrow$$

$$\Rightarrow -2 \cdot z = \frac{1}{2} - \frac{4}{1} \Rightarrow -2 \cdot z = \frac{1-8}{2} \Rightarrow -2 \cdot z = -\frac{7}{2} \Rightarrow -2 \cdot z = -\frac{7}{2} \,/ \cdot \left(-\frac{1}{2}\right) \Rightarrow z = \frac{7}{4}.$$

Računamo y.

$$\left.\begin{array}{l} x = 2 \ , \ z = \frac{7}{4} \ , \ v = \frac{19}{12} \\ x + y - z - v = \frac{1}{3} \end{array}\right\} \Rightarrow 2 + y - \frac{7}{4} - \frac{19}{12} = \frac{1}{3} \Rightarrow y = \frac{1}{3} - 2 + \frac{7}{4} + \frac{19}{12} \Rightarrow$$

$$\Rightarrow y = \frac{1}{3} - \frac{2}{1} + \frac{7}{4} + \frac{19}{12} \Rightarrow y = \frac{4 - 24 + 21 + 19}{12} \Rightarrow y = \frac{20}{12} \Rightarrow y = \frac{20}{12} \Rightarrow y = \frac{5}{3}.$$

Rješenje sustava je

$$(x, \ y, \ z, \ v) = \left(2, \ \frac{5}{3}, \ \frac{7}{4}, \ \frac{19}{12}\right).$$

Vježba 144

Odmor!

Rezultat: …

Zadatak 145 (Maturant, ekonomska škola)

Riješite sustav jednadžba: $\frac{x \cdot y}{x + y} = \frac{1}{5} \quad , \quad \frac{x \cdot y}{x - y} = 1.$

Rješenje 145

Ponovimo!

$$n = \frac{n}{1} \quad , \quad \frac{a}{b} = \frac{c}{d} \Rightarrow \frac{b}{a} = \frac{d}{c} \quad , \quad \frac{a+b}{n} = \frac{a}{n} + \frac{b}{n} \quad , \quad \frac{a-b}{n} = \frac{a}{n} - \frac{b}{n}.$$

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \ , \ n \neq 0 \ , \ n \neq 1.$$

Sustav možemo pisati u obliku:

$$\left.\begin{array}{l} \frac{x \cdot y}{x + y} = \frac{1}{5} \\ \frac{x \cdot y}{x - y} = 1 \end{array}\right\} \Rightarrow \left.\begin{array}{l} \frac{x + y}{x \cdot y} = 5 \\ \frac{x - y}{x \cdot y} = 1 \end{array}\right\} \Rightarrow \left.\begin{array}{l} \frac{x}{x \cdot y} + \frac{y}{x \cdot y} = 5 \\ \frac{x}{x \cdot y} - \frac{y}{x \cdot y} = 1 \end{array}\right\} \Rightarrow \left.\begin{array}{l} \frac{x}{x \cdot y} + \frac{y}{x \cdot y} = 5 \\ \frac{x}{x \cdot y} - \frac{y}{x \cdot y} = 1 \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} \frac{1}{y} + \frac{1}{x} = 5 \\ \frac{1}{y} - \frac{1}{x} = 1 \end{array}\right\} \Rightarrow \left[\begin{array}{l} \text{metoda suprotnih} \\ \text{koeficijenata} \end{array}\right] \Rightarrow \frac{2}{y} = 6 \Rightarrow \frac{2}{y} = 6 \,/ \cdot \frac{1}{2} \Rightarrow \frac{1}{y} = 3 \Rightarrow$$

$$\Rightarrow \frac{1}{y} = \frac{3}{1} \Rightarrow y = \frac{1}{3}.$$

Računamo x.

$$\left.\begin{array}{l}\frac{1}{y}=3\\ \frac{1}{y}+\frac{1}{x}=5\end{array}\right\}\Rightarrow 3+\frac{1}{x}=5\Rightarrow \frac{1}{x}=5-3\Rightarrow \frac{1}{x}=2\Rightarrow \frac{1}{x}=\frac{2}{1}\Rightarrow x=\frac{1}{2}.$$

Rezultat:

$$(x,\ y)=\left(\frac{1}{2},\ \frac{1}{3}\right).$$

Vježba 145

Odmor!

Rezultat: …

Zadatak 146 (Maturantica, ekonomska škola)

Riješite sustav jednadžba: $x=\frac{x+1}{2}-\frac{y-3}{4}$, $y=\frac{x+1}{3}+\frac{y-3}{2}.$

Rješenje 146

Ponovimo!

$$n=\frac{n}{1}\quad,\quad \frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d}\quad,\quad \frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

$$\left.\begin{array}{l}x=\frac{x+1}{2}-\frac{y-3}{4}\\ y=\frac{x+1}{3}+\frac{y-3}{2}\end{array}\right\}\Rightarrow \left.\begin{array}{l}x=\frac{x+1}{2}-\frac{y-3}{4}\ /\cdot 4\\ y=\frac{x+1}{3}+\frac{y-3}{2}\ /\cdot 6\end{array}\right\}\Rightarrow \left.\begin{array}{l}4\cdot x=2\cdot(x+1)-1\cdot(y-3)\\ 6\cdot y=2\cdot(x+1)+3\cdot(y-3)\end{array}\right\}\Rightarrow$$

$$\Rightarrow \left.\begin{array}{l}4\cdot x=2\cdot x+2-y+3\\ 6\cdot y=2\cdot x+2+3\cdot y-9\end{array}\right\}\Rightarrow \left.\begin{array}{l}4\cdot x-2\cdot x+y=2+3\\ 6\cdot y-2\cdot x-3\cdot y=2-9\end{array}\right\}\Rightarrow \left.\begin{array}{l}2\cdot x+y=5\\ -2\cdot x+3\cdot y=-7\end{array}\right\}\Rightarrow$$

$$\Rightarrow \left[\begin{array}{l}\text{metoda suprotnih}\\ \text{koeficijenata}\end{array}\right]\Rightarrow 4\cdot y=-2\Rightarrow 4\cdot y=-2\ /\cdot\frac{1}{4}\Rightarrow y=-\frac{2}{4}\Rightarrow y=-\frac{2}{4}\Rightarrow y=-\frac{1}{2}.$$

Računamo x.

$$\left.\begin{array}{l}y=-\frac{1}{2}\\ 2\cdot x+y=5\end{array}\right\}\Rightarrow 2\cdot x-\frac{1}{2}=5\Rightarrow 2\cdot x=5+\frac{1}{2}\Rightarrow 2\cdot x=\frac{5}{1}+\frac{1}{2}\Rightarrow 2\cdot x=\frac{10+1}{2}\Rightarrow$$

$$\Rightarrow 2\cdot x=\frac{11}{2}\Rightarrow 2\cdot x=\frac{11}{2}\ /\cdot\frac{1}{2}\Rightarrow x=\frac{11}{4}.$$

Rezultat: $(x,\ y)=\left(\frac{11}{4},\ -\frac{1}{2}\right).$

Vježba 146

Odmor!

Rezultat: …

Zadatak 147 (Ivan, strukovna)

Marko će 3 jogurta i 6 peciva platiti 26.25 kn, a 4 jogurta i 4 peciva 25 kn. Kolika je cijena jednoga jogurta?

$$A.\ 1.25\ kn \qquad B.\ 3.75\ kn \qquad C.\ 5.72\ kn \qquad D.\ 7.32\ kn$$

Rješenje 147

Ponovimo!

Prirodni je broj djeljiv s 3 ako mu je zbroj znamenki djeljiv s 3.

Neka je:

- x cijena jednoga jogurta
- y cijena jednoga peciva.





Rečenicu "Marko će 3 jogurta i 6 peciva platiti 26.25 kn, …" zapišimo u obliku jednadžbe:

$$3\cdot x+6\cdot y=26.25 \;\Rightarrow\; 3\cdot x+6\cdot y=26.25 \;/:3 \;\Rightarrow\; x+2\cdot y=8.75.$$

Rečenicu "…, a 4 jogurta i 4 peciva 25 kn." zapišimo u obliku jednadžbe:

$$4\cdot x+4\cdot y=25.$$

Riješimo sustav jednadžba:

$$\left.\begin{array}{l} x+2\cdot y=8.75 \\ 4\cdot x+4\cdot y=25 \end{array}\right\} \Rightarrow \left[\begin{array}{l} \text{metoda suprotnih} \\ \text{koeficijenata} \end{array}\right] \Rightarrow \left.\begin{array}{l} x+2\cdot y=8.75 \;/\cdot(-2) \\ 4\cdot x+4\cdot y=25 \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} -2\cdot x-4\cdot y=-17.50 \\ 4\cdot x+4\cdot y=25 \end{array}\right\} \Rightarrow 2\cdot x=7.50 \Rightarrow 2\cdot x=7.50 \;/:2 \;\Rightarrow\; x=3.75\ kn.$$

Odgovor je pod B.

Vježba 147

Marko će 3 jogurta i 6 peciva platiti 26.25 kn, a 4 jogurta i 4 peciva 25 kn. Kolika je cijena jednoga peciva?

$$A.\ 2.50\ kn \qquad B.\ 2.75\ kn \qquad C.\ 2.80\ kn \qquad D.\ 2.95\ kn$$

Rezultat: A.

Zadatak 148 (Marko, maturant gimnazije)

Za koje vrijednosti parametra $a\in R$ rješenje sustava $\begin{cases} -2\cdot x+y=a^2-1 \\ 3\cdot x+2\cdot y=2\cdot a^2+7\cdot a+5 \end{cases}$

zadovoljava uvjet y < x + 2?

$$A.\ [-2,1] \qquad B.\ \langle -2,1\rangle \qquad C.\ \langle 1,2\rangle \qquad D.\ [-2,0]$$

Rješenje 148

Ponovimo!

$$a^2-b^2=(a-b)\cdot(a+b) \quad , \quad a^1=a \quad , \quad a^n\cdot a^m=a^{n+m}.$$

$$a\cdot b<0 \Rightarrow \left.\begin{array}{l} a>0 \\ b<0 \end{array}\right\} \text{ ili } \left.\begin{array}{l} a<0 \\ b>0 \end{array}\right\}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Interval je skup svih realnih brojeva x sadržanih između brojeva a i b koji na brojevnom pravcu ispunjava dužinu od a do b. Brojevi a i b nazivaju se granice intervala.

Zatvoreni interval, $[a, b]$, sadržava granice: $a \le x \le b$. Često se naziva i segment.

Otvoreni interval, $\langle a, b\rangle$ ili (a, b), ne sadržava granice: $a < x < b$.

Skup zadajemo nabrajanjem njegovih elemenata ili opisom karakterističnih svojstava koja posjeduju njegovi elementi.

Presjek skupova A i B je skup koji sadrži sve elemente koji se nalaze i u skupu A i u skupu B. Označavamo ga $A \cap B$.

Neka je U univerzalni skup, te A i B proizvoljni skupovi koji su podskupovi skupa U. Tada je:

$$A \cap B = \{ x \in U : x \in A \text{ i } x \in B \}.$$

Riješimo sustav.

$$\left.\begin{array}{l} -2\cdot x + y = a^2 - 1 \\ 3\cdot x + 2\cdot y = 2\cdot a^2 + 7\cdot a + 5 \end{array}\right\} \Rightarrow \left[\begin{array}{l} \text{metoda suprotnih} \\ \text{koeficijenata} \end{array}\right] \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} -2\cdot x + y = a^2 - 1 \;/\cdot(-2) \\ 3\cdot x + 2\cdot y = 2\cdot a^2 + 7\cdot a + 5 \end{array}\right\} \Rightarrow \left.\begin{array}{l} 4\cdot x - 2\cdot y = -2\cdot a^2 + 2 \\ 3\cdot x + 2\cdot y = 2\cdot a^2 + 7\cdot a + 5 \end{array}\right\} \Rightarrow 7\cdot x = 7\cdot a + 7 \Rightarrow$$

$$\Rightarrow 7\cdot x = 7\cdot(a+1) \Rightarrow 7\cdot x = 7\cdot(a+1) \;/:7 \Rightarrow x = a + 1.$$

Računamo y.

$$\left.\begin{array}{l} x = a + 1 \\ -2\cdot x + y = a^2 - 1 \end{array}\right\} \Rightarrow -2\cdot(a+1) + y = a^2 - 1 \Rightarrow -2\cdot a - 2 + y = a^2 - 1 \Rightarrow y = a^2 - 1 + 2\cdot a + 2 \Rightarrow$$

$$\Rightarrow y = a^2 + 2\cdot a + 1.$$

Iz uvjeta zadatka slijedi:

$$y < x + 2 \Rightarrow a^2 + 2\cdot a + 1 < a + 1 + 2 \Rightarrow a^2 + 2\cdot a + 1 - a - 1 - 2 < 0 \Rightarrow a^2 + 2\cdot a + 1 - a - 1 - 2 < 0 \Rightarrow$$

$$\Rightarrow a^2 + a - 2 < 0.$$

Rastavimo trinom $a^2 + a - 2$ na faktore! Pokažimo dva načina (ima ih još, ali treba biti umjeren 😊).

1.inačica

$$a^2 + a - 2 = a^2 - 1 + a - 1 = \left(a^2 - 1\right) + (a-1) = (a-1)\cdot(a+1) + (a-1) =$$

$$= (a-1)\cdot(a+1+1) = (a-1)\cdot(a+2).$$

2.inačica

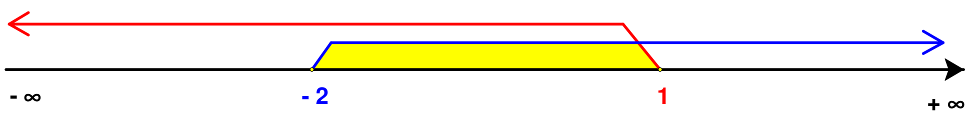
$$a^2 + a - 2 = a^2 + 2\cdot a - a - 2 = \left(a^2 + 2\cdot a\right) + (-a - 2) = a\cdot(a+2) - (a+2) = (a+2)\cdot(a-1).$$

Sada možemo riješiti nejednadžbu.

$$a^2 + a - 2 < 0 \Rightarrow (a-1)\cdot(a+2) < 0 \Rightarrow \left.\begin{array}{l} \text{1.slučaj:} a - 1 < 0 \;,\; a + 2 > 0 \\ \text{2.slučaj:} a - 1 > 0 \;,\; a + 2 < 0 \end{array}\right\}.$$

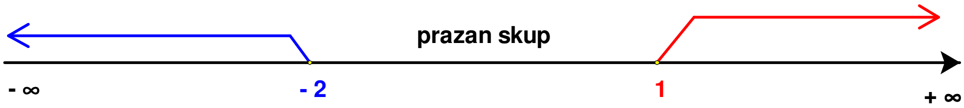
1.slučaj

$$(a-1)\cdot(a+2) < 0 \Rightarrow \left.\begin{array}{l} a - 1 < 0 \\ a + 2 > 0 \end{array}\right\} \Rightarrow \left.\begin{array}{l} a < 1 \\ a > -2 \end{array}\right\} \Rightarrow \left.\begin{array}{l} a \in \langle -\infty, 1\rangle \\ a \in \langle -2, +\infty\rangle \end{array}\right\} \Rightarrow \left[\begin{array}{l} \text{presjek} \\ \text{skupova} \end{array}\right] \Rightarrow a \in \langle -2, 1\rangle.$$



2.slučaj

$$(a-1)\cdot(a+2) < 0 \Rightarrow \left.\begin{array}{l} a - 1 > 0 \\ a + 2 < 0 \end{array}\right\} \Rightarrow \left.\begin{array}{l} a > 1 \\ a < -2 \end{array}\right\} \Rightarrow \left.\begin{array}{l} a \in \langle 1, +\infty\rangle \\ a \in \langle -\infty, -2\rangle \end{array}\right\} \Rightarrow \left[\begin{array}{l} \text{presjek} \\ \text{skupova} \end{array}\right] \Rightarrow a \in \emptyset.$$



Odgovor je pod B.

Vježba 148

Odmor!

Rezultat: …

Zadatak 149 (Tomislav, gimnazija)

Riješite sustav jednadžba: $\begin{cases} x^2+5\cdot y^2+4\cdot z^2+4\cdot x\cdot y+4\cdot y\cdot z=125 \\ x^2+3\cdot y^2-4\cdot z^2+4\cdot x\cdot y-4\cdot y\cdot z=75 \end{cases}$, $x,\ y,\ z\in N.$

Rješenje 149

Ponovimo!

$$\left.\begin{matrix} a=b \\ c=d \end{matrix}\right\} \Rightarrow a+c=b+d \quad , \quad \left.\begin{matrix} a=b \\ c=d \end{matrix}\right\} \Rightarrow a-c=b-d \quad , \quad (a+b)^2=a^2+2\cdot a\cdot b+b^2.$$

$$(a-b)^2=a^2-2\cdot a\cdot b+b^2 \quad , \quad \sqrt{a^2}=a\ ,\ a\geq \quad , \quad (a\cdot b)^n=a^n\cdot b^n.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Jednadžbe zbrojimo.

$$\left.\begin{matrix} x^2+5\cdot y^2+4\cdot z^2+4\cdot x\cdot y+4\cdot y\cdot z=125 \\ x^2+3\cdot y^2-4\cdot z^2+4\cdot x\cdot y-4\cdot y\cdot z=75 \end{matrix}\right\} \Rightarrow \begin{bmatrix} \text{zbrojimo} \\ \text{jednadžbe} \end{bmatrix} \Rightarrow$$

$$\Rightarrow x^2+5\cdot y^2+4\cdot z^2+4\cdot x\cdot y+4\cdot y\cdot z+x^2+3\cdot y^2-4\cdot z^2+4\cdot x\cdot y-4\cdot y\cdot z=125+75 \Rightarrow$$

$$\Rightarrow x^2+5\cdot y^2+4\cdot z^2+4\cdot x\cdot y+4\cdot y\cdot z+x^2+3\cdot y^2-4\cdot z^2+4\cdot x\cdot y-4\cdot y\cdot z=200 \Rightarrow$$

$$\Rightarrow x^2+5\cdot y^2+4\cdot x\cdot y+x^2+3\cdot y^2+4\cdot x\cdot y=200 \Rightarrow 2\cdot x^2+8\cdot y^2+8\cdot x\cdot y=200 \Rightarrow$$

$$\Rightarrow 2\cdot x^2+8\cdot y^2+8\cdot x\cdot y=200\ /:2 \Rightarrow x^2+4\cdot y^2+4\cdot x\cdot y=100 \Rightarrow x^2+4\cdot x\cdot y+4\cdot y^2=100 \Rightarrow$$

$$\Rightarrow x^2+2\cdot x\cdot 2\cdot y+(2\cdot y)^2=100 \Rightarrow (x+2\cdot y)^2=100 \Rightarrow (x+2\cdot y)^2=100\ /\sqrt{\ } \Rightarrow$$

$$\Rightarrow x+2\cdot y=10.$$

Jednadžbe oduzmemo.

$$\left.\begin{matrix} x^2+5\cdot y^2+4\cdot z^2+4\cdot x\cdot y+4\cdot y\cdot z=125 \\ x^2+3\cdot y^2-4\cdot z^2+4\cdot x\cdot y-4\cdot y\cdot z=75 \end{matrix}\right\} \Rightarrow \begin{bmatrix} \text{oduzmemo} \\ \text{jednadžbe} \end{bmatrix} \Rightarrow$$

$$\Rightarrow x^2+5\cdot y^2+4\cdot z^2+4\cdot x\cdot y+4\cdot y\cdot z-\left(x^2+3\cdot y^2-4\cdot z^2+4\cdot x\cdot y-4\cdot y\cdot z\right)=125-75 \Rightarrow$$

$$\Rightarrow x^2+5\cdot y^2+4\cdot z^2+4\cdot x\cdot y+4\cdot y\cdot z-x^2-3\cdot y^2+4\cdot z^2-4\cdot x\cdot y+4\cdot y\cdot z=50 \Rightarrow$$

$$\Rightarrow x^2+5\cdot y^2+4\cdot z^2+4\cdot x\cdot y+4\cdot y\cdot z-x^2-3\cdot y^2+4\cdot z^2-4\cdot x\cdot y+4\cdot y\cdot z=50 \Rightarrow$$

$$\Rightarrow 5\cdot y^2+4\cdot z^2+4\cdot y\cdot z-3\cdot y^2+4\cdot z^2+4\cdot y\cdot z=50 \Rightarrow$$

$$\Rightarrow 2\cdot y^2+8\cdot y\cdot z+8\cdot z^2=50 \Rightarrow 2\cdot y^2+8\cdot y\cdot z+8\cdot z^2=50\ /:2 \Rightarrow y^2+4\cdot y\cdot z+4\cdot z^2=25 \Rightarrow$$

$$\Rightarrow y^2+2\cdot y\cdot 2\cdot z+(2\cdot z)^2=25 \Rightarrow (y+2\cdot z)^2=25 \Rightarrow (y+2\cdot z)^2=25 \ /\sqrt{} \Rightarrow$$

$$\Rightarrow y+2\cdot z=5.$$

Dobili smo sustav od dvije jednadžbe s tri nepoznanice. Dvije nepoznanice izračunamo pomoću treće.

$$\left.\begin{matrix} x+2\cdot y=10 \\ y+2\cdot z=5 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x=10-2\cdot y \\ y=5-2\cdot z \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x=10-2\cdot(5-2\cdot z) \\ y=5-2\cdot z \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x=10-10+4\cdot z \\ y=5-2\cdot z \end{matrix}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{matrix} x=10-10+4\cdot z \\ y=5-2\cdot z \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x=4\cdot z \\ y=5-2\cdot z \end{matrix}\right\}.$$

Budući da su x, y, z prirodni brojevi dobije se:

- $\left.\begin{matrix} x=4\cdot z \\ y=5-2\cdot z \end{matrix}\right\} \Rightarrow [z=1] \Rightarrow \left.\begin{matrix} x=4\cdot 1 \\ y=5-2\cdot 1 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x=4 \\ y=3 \end{matrix}\right\} \Rightarrow (x, y, z)=(4, 3, 1).$
- $\left.\begin{matrix} x=4\cdot z \\ y=5-2\cdot z \end{matrix}\right\} \Rightarrow [z=2] \Rightarrow \left.\begin{matrix} x=4\cdot 2 \\ y=5-2\cdot 2 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x=8 \\ y=1 \end{matrix}\right\} \Rightarrow (x, y, z)=(8, 1, 2).$

Vježba 149

Odmor!

Rezultat: …

Zadatak 150 (Ivica, maturant)

Čemu je jednak y u rješenju sustava jednadžba $\begin{cases} 3\cdot x-25\cdot y=-57.6 \\ \frac{y}{3}-x=0 \end{cases}$?

A. 0.9 *B.* 1.6 *C.* 2.4 *D.* 3.2

Rješenje 150

Ponovimo!

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \ , \ n\neq 0 \ , \ n\neq 1.$$

$$\left.\begin{matrix} 3\cdot x-25\cdot y=-57.6 \\ \frac{y}{3}-x=0 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} 3\cdot x-25\cdot y=-57.6 \\ \frac{y}{3}=x \end{matrix}\right\} \Rightarrow \left.\begin{matrix} 3\cdot x-25\cdot y=-57.6 \\ x=\frac{y}{3} \end{matrix}\right\} \Rightarrow \left[\begin{matrix} \text{metoda} \\ \text{zamjene} \end{matrix}\right] \Rightarrow$$

$$\Rightarrow 3\cdot\frac{y}{3}-25\cdot y=-57.6 \Rightarrow 3\cdot\frac{y}{3}-25\cdot y=-57.6 \Rightarrow y-25\cdot y=-57.6 \Rightarrow -24\cdot y=-57.6 \Rightarrow$$

$$\Rightarrow -24\cdot y=-57.6 \ /:(-24) \Rightarrow y=2.4.$$

Odgovor je pod C.

Vježba 150

Čemu je jednak x u rješenju sustava jednadžba $\begin{cases} 3\cdot x-25\cdot y=-57.6 \\ \frac{y}{3}-x=0 \end{cases}$?

A. 0.7 *B.* 0.6 *C.* 0.8 *D.* 0.9

Rezultat: C.

Zadatak 151 (Danijel, maturant)

Koliki mora biti parametar m da sustav jednadžba: $\begin{cases} m \cdot x + 3 \cdot y = 1 \\ 2 \cdot x - \sqrt{3} \cdot y = 7 \end{cases}$ ne bi imao rješenja?

$A.\ m = 2 \cdot \sqrt{3}$ $\quad B.\ m = 3$ $\quad C.\ m = -3$ $\quad D.\ m = -2 \cdot \sqrt{3}$

Rješenje 151

Ponovimo!

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d} \quad , \quad a^1 = a \quad , \quad a^n \cdot a^m = a^{n+m} \quad , \quad \left(\sqrt{a}\right)^2 = a.$$

Parametar
Vladimir Anić, Ivo Goldstein, Rječnik stranih riječi, Novi Liber, Zagreb, 2002.
Veličina, obično realna varijabla, čije vrijednosti služe za razlikovanje elemenata nekog skupa točaka funkcija, jednadžbi ili drugih matematičkih objekata.
Bratoljub Klaić, Rječnik stranih riječi, Nakladni zavod MH, Zagreb, 1983.
Veličina o kojoj ovisi funkcija ili oblik krivulje.
Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b + c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b + c).$$

Proširiti razlomak znači brojnik i nazivnik tog razlomka pomnožiti istim brojem različitim od nule i jedinice

$$\frac{a}{b} = \frac{a \cdot n}{b \cdot n} \ , \ n \neq 0 \ , \ n \neq 1.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \ , \ n \neq 0 \ , \ n \neq 1.$$

S nulom se ne smije dijeliti.
Grafički riješiti sustav jednadžba $a_1 \cdot x + b_1 \cdot y = c_1$ i $a_2 \cdot x + b_2 \cdot y = c_2$ znači crtanjem odrediti presjek dvaju pravaca (ako postoji) pri čemu su jednadžbe pravaca $y = -\frac{a_1}{b_1} \cdot x + \frac{c_1}{b_1}$ i $y = -\frac{a_2}{b_2} \cdot x + \frac{c_2}{b_2}$.

Ako su pravci usporedni, sustav nema rješenja.
Jednadžba pravca oblika

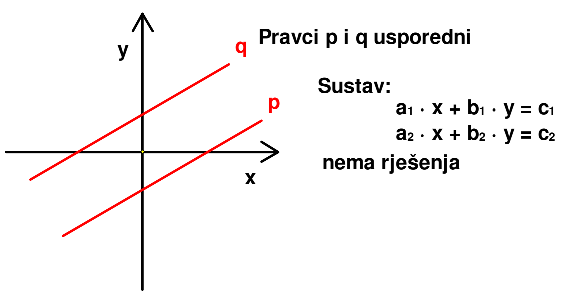
$$A \cdot x + B \cdot y + C = 0$$

naziva se implicitni oblik jednadžbe pravca ili kraće, opći oblik jednadžbe pravca.
Jednadžba pravca oblika

$$y = k \cdot x + l$$

naziva se eksplicitni oblik jednadžbe pravca ili kraće, eksplicitna jednadžba pravca. Broj k naziva se koeficijent smjera pravca. Broj l nazivamo odsječak pravca na osi y.
Uvjet usporednosti (paralelnosti):
Ako su pravci dani eksplicitnim jednadžbama $y = k_1 \cdot x + l_1$, $y = k_2 \cdot x + l_2$, tada su usporedni ako i samo ako je

$$k_1 = k_2.$$



1.inačica

Riješimo sustav jednadžba.

$$\left.\begin{array}{l} m\cdot x+3\cdot y=1 \\ 2\cdot x-\sqrt{3}\cdot y=7 \end{array}\right\} \Rightarrow \left[\begin{array}{l} \text{metoda suprotnih} \\ \text{koeficijenata} \end{array}\right] \Rightarrow \left.\begin{array}{l} m\cdot x+3\cdot y=1 \;/\cdot 2 \\ 2\cdot x-\sqrt{3}\cdot y=7 \;/\cdot(-m) \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} 2\cdot m\cdot x+6\cdot y=2 \\ -2\cdot m\cdot x+\sqrt{3}\cdot m\cdot y=-7\cdot m \end{array}\right\} \Rightarrow 6\cdot y+\sqrt{3}\cdot m\cdot y=2-7\cdot m \Rightarrow \left(6+\sqrt{3}\cdot m\right)\cdot y=2-7\cdot m.$$

Ako je $6+\sqrt{3}\cdot m=0$ slijedi

$$6+\sqrt{3}\cdot m=0 \Rightarrow \sqrt{3}\cdot m=-6 \Rightarrow \sqrt{3}\cdot m=-6 \;/\cdot\frac{1}{\sqrt{3}} \Rightarrow m=-\frac{6}{\sqrt{3}} \Rightarrow m=-\frac{6}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} \Rightarrow$$

$$\Rightarrow m=-\frac{6\cdot\sqrt{3}}{\left(\sqrt{3}\right)^2} \Rightarrow m=-\frac{6\cdot\sqrt{3}}{3} \Rightarrow m=-\frac{6\cdot\sqrt{3}}{3} \Rightarrow m=-2\cdot\sqrt{3}.$$

Onda je

$$0\cdot y=2-7\cdot\left(-2\cdot\sqrt{3}\right) \Rightarrow 0=2+14\cdot\sqrt{3}$$

pa jednadžba nema rješenja.

Odgovor je pod D.

2.inačica

Jednadžbe pravaca napišemo u eksplicitnom obliku kako bismo mogli odrediti njihove koeficijente smjerova.

$$\left.\begin{array}{l} m\cdot x+3\cdot y=1 \\ 2\cdot x-\sqrt{3}\cdot y=7 \end{array}\right\} \Rightarrow \left.\begin{array}{l} 3\cdot y=-m\cdot x+1 \\ -\sqrt{3}\cdot y=-2\cdot x+7 \end{array}\right\} \Rightarrow \left.\begin{array}{l} 3\cdot y=-m\cdot x+1 \;/\cdot\frac{1}{3} \\ -\sqrt{3}\cdot y=-2\cdot x+7 \;/\cdot\left(-\frac{1}{\sqrt{3}}\right) \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} y=-\frac{m}{3}\cdot x+\frac{1}{3} \\ y=\frac{2}{\sqrt{3}}\cdot x-\frac{7}{\sqrt{3}} \end{array}\right\} \Rightarrow \left.\begin{array}{l} k_1=-\frac{m}{3} \\ k_2=\frac{2}{\sqrt{3}} \end{array}\right\}.$$

Budući da zadani sustav ne smije imati rješenja, pravci moraju biti usporedni kako ne bi imali niti jednu točku zajedničku. Njihovi koeficijenti smjera moraju biti jednaki.

$$k_1=k_2 \Rightarrow -\frac{m}{3}=\frac{2}{\sqrt{3}} \Rightarrow -\frac{m}{3}=\frac{2}{\sqrt{3}} \;/\cdot(-3) \Rightarrow m=-\frac{6}{\sqrt{3}} \Rightarrow m=-\frac{6}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} \Rightarrow$$

$$\Rightarrow m=-\frac{6\cdot\sqrt{3}}{\left(\sqrt{3}\right)^2} \Rightarrow m=-\frac{6\cdot\sqrt{3}}{3} \Rightarrow m=-\frac{6\cdot\sqrt{3}}{3} \Rightarrow m=-2\cdot\sqrt{3}.$$

Odgovor je pod D.

Vježba 151

Odmor!

Rezultat: …

Zadatak 152 (Davor, maturant)

U koliko se točaka sijeku krivulje $y = 2^{\frac{|x|}{x}}$ i $y = x + \frac{3}{2}$?

$A.\ 0 \qquad B.\ 1 \qquad C.\ 2 \qquad D.\ 3$

Rješenje 152

Ponovimo!

$$a^1 = a \quad , \quad a^{-n} = \frac{1}{a^n} \quad , \quad a - \frac{b}{c} = \frac{a \cdot c - b}{c} \quad , \quad \frac{a}{n} - \frac{b}{n} = \frac{a-b}{n}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \ , \ n \neq 0 \ , \ n \neq 1.$$

Za realni broj x njegova je apsolutna vrijednost (modul) broj $|x|$ koji određujemo na ovaj način:

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0. \end{cases}$$

Ako je broj x pozitivan ili nula, tada je on jednak svojoj apsolutnoj vrijednosti. Za svaki x, x ≥ 0, vrijedi $|x| = x$.
Ako je x negativan broj, njegova apsolutna vrijednost je suprotan broj – x koji je pozitivan. Za svaki x, x < 0, je $|x| = -x$.

Prvi slučaj

$$x > 0 \ \Rightarrow \ |x| = x.$$

Tada je

$$\left.\begin{matrix} y = 2^{\frac{|x|}{x}} \\ y = x + \frac{3}{2} \end{matrix}\right\} \Rightarrow \left.\begin{matrix} y = 2^{\frac{x}{x}} \\ y = x + \frac{3}{2} \end{matrix}\right\} \Rightarrow \left.\begin{matrix} y = 2^{\frac{x}{x}} \\ y = x + \frac{3}{2} \end{matrix}\right\} \Rightarrow \left.\begin{matrix} y = 2^1 \\ y = x + \frac{3}{2} \end{matrix}\right\} \Rightarrow \left.\begin{matrix} y = 2 \\ y = x + \frac{3}{2} \end{matrix}\right\} \Rightarrow \left[\begin{matrix} \text{metoda} \\ \text{komparacije} \end{matrix}\right] \Rightarrow$$

$$\Rightarrow x + \frac{3}{2} = 2 \ \Rightarrow \ x = 2 - \frac{3}{2} \ \Rightarrow \ x = \frac{1}{2}.$$

Sjecište ima koordinate:

$$S_1(x, y) = S_1\left(\frac{1}{2}, 2\right).$$

Drugi slučaj

$$x < 0 \ \Rightarrow \ |x| = -x.$$

Tada je

$$\left.\begin{matrix} y = 2^{\frac{|x|}{x}} \\ y = x + \frac{3}{2} \end{matrix}\right\} \Rightarrow \left.\begin{matrix} y = 2^{\frac{-x}{x}} \\ y = x + \frac{3}{2} \end{matrix}\right\} \Rightarrow \left.\begin{matrix} y = 2^{\frac{-x}{x}} \\ y = x + \frac{3}{2} \end{matrix}\right\} \Rightarrow \left.\begin{matrix} y = 2^{-1} \\ y = x + \frac{3}{2} \end{matrix}\right\} \Rightarrow \left.\begin{matrix} y = \frac{1}{2} \\ y = x + \frac{3}{2} \end{matrix}\right\} \Rightarrow \left[\begin{matrix} \text{metoda} \\ \text{komparacije} \end{matrix}\right] \Rightarrow$$

$$\Rightarrow x + \frac{3}{2} = \frac{1}{2} \ \Rightarrow \ x = \frac{1}{2} - \frac{3}{2} \ \Rightarrow \ x = \frac{1-3}{2} \ \Rightarrow \ x = -\frac{2}{2} \ \Rightarrow \ x = -\frac{2}{2} \ \Rightarrow \ x = -1.$$

Sjecište ima koordinate:

$$S_2(x, y) = S_2\left(-1, \frac{1}{2}\right).$$

Krivulje imaju dva sjecišta.
Odgovor je pod C.

Vježba 152

U koliko se točaka sijeku krivulje $y=2^{\frac{|x|}{x}}$ i $y=x+\frac{1}{2}$?

$A.\ 0 \qquad B.\ 1 \qquad C.\ 2 \qquad D.\ 3$

Rezultat: C.

Zadatak 153 (Vinko, maturant)

Riješite sustav s tri nepoznanice: $\begin{cases} x+y=7 \\ z-y=1 \\ 3\cdot x-z=4 \end{cases}$.

Rješenje 153

Ponovimo!

$$\left.\begin{matrix} a=b \\ c=d \end{matrix}\right\} \Rightarrow a+c=b+d.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

1.inačica

$$\left.\begin{matrix} x+y=7 \\ z-y=1 \\ 3\cdot x-z=4 \end{matrix}\right\} \Rightarrow \begin{bmatrix} \text{zbrojimo} \\ \text{jednadžbe} \end{bmatrix} \Rightarrow x+y+z-y+3\cdot x-z=7+1+4 \Rightarrow$$

$$\Rightarrow x+y+z-y+3\cdot x-z=12 \Rightarrow 4\cdot x=12 \Rightarrow 4\cdot x=12\ /:4 \Rightarrow x=3.$$

Računamo y.

$$\left.\begin{matrix} x=3 \\ x+y=7 \end{matrix}\right\} \Rightarrow 3+y=7 \Rightarrow y=7-3 \Rightarrow y=4.$$

Računamo z.

$$\left.\begin{matrix} y=4 \\ z-y=1 \end{matrix}\right\} \Rightarrow z-4=1 \Rightarrow z=1+4 \Rightarrow z=5.$$

Rješenje je uređena trojka

$$(x,\ y,\ z)=(3,\ 4,\ 5).$$

2.inačica

$$\left.\begin{matrix} x+y=7 \\ z-y=1 \\ 3\cdot x-z=4 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x=7-y \\ z-y=1 \\ 3\cdot x-z=4 \end{matrix}\right\} \Rightarrow \begin{bmatrix} \text{metoda} \\ \text{zamjene} \end{bmatrix} \Rightarrow \left.\begin{matrix} z-y=1 \\ 3\cdot(7-y)-z=4 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} z-y=1 \\ 21-3\cdot y-z=4 \end{matrix}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{matrix} z=1+y \\ -3\cdot y-z=4-21 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} z=1+y \\ -3\cdot y-z=-17 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} z=1+y \\ -3\cdot y-z=-17\ /\cdot(-1) \end{matrix}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{matrix} z=1+y \\ 3\cdot y+z=17 \end{matrix}\right\} \Rightarrow \begin{bmatrix} \text{metoda} \\ \text{zamjene} \end{bmatrix} \Rightarrow 3\cdot y+1+y=17 \Rightarrow 3\cdot y+y=17-1 \Rightarrow 4\cdot y=16 \Rightarrow$$

$$\Rightarrow 4\cdot y=16\ /:4 \Rightarrow y=4.$$

Računamo x.

$$\left.\begin{matrix} y=4 \\ x+y=7 \end{matrix}\right\} \Rightarrow x+4=7 \Rightarrow x=7-4 \Rightarrow x=3.$$

Računamo z.

$$\left.\begin{array}{l} y=4 \\ z-y=1 \end{array}\right\} \Rightarrow z-4=1 \Rightarrow z=1+4 \Rightarrow z=5.$$

Rješenje je uređena trojka

$$(x, y, z)=(3, 4, 5).$$

3.inačica

$$\left.\begin{array}{l} x+y=7 \\ z-y=1 \\ 3\cdot x-z=4 \end{array}\right\} \Rightarrow \left.\begin{array}{l} x=7-y \\ z=1+y \\ 3\cdot x-z=4 \end{array}\right\} \Rightarrow \left[\begin{array}{l} \text{metoda} \\ \text{zamjene} \end{array}\right] \Rightarrow 3\cdot(7-y)-(1+y)=4 \Rightarrow$$

$$\Rightarrow 21-3\cdot y-1-y=4 \Rightarrow -3\cdot y-y=4-21+1 \Rightarrow -4\cdot y=-16 \Rightarrow -4\cdot y=-16 \; /:(-4) \Rightarrow$$

$$\Rightarrow y=4.$$

Računamo x.

$$\left.\begin{array}{l} y=4 \\ x+y=7 \end{array}\right\} \Rightarrow x+4=7 \Rightarrow x=7-4 \Rightarrow x=3.$$

Računamo z.

$$\left.\begin{array}{l} y=4 \\ z-y=1 \end{array}\right\} \Rightarrow z-4=1 \Rightarrow z=1+4 \Rightarrow z=5.$$

Rješenje je uređena trojka

$$(x, y, z)=(3, 4, 5).$$

Vježba 153

Odmor!

Rezultat: …

Zadatak 154 (Filip, gimnazija)

Riješite sustav: $\left\{\begin{array}{l} \frac{x^2}{y}+\frac{y^2}{x}=\frac{91}{12} \\ x+y=7 \end{array}\right.$.

Rješenje 154

Ponovimo!

$$a^1=a \quad , \quad a^n\cdot a^m=a^{n+m} \quad , \quad a^3+b^3=(a+b)\cdot\left(a^2-a\cdot b+b^2\right).$$

$$(a-b)^2=a^2-2\cdot a\cdot b+b^2.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \; , \; n\neq 0 \; , \; n\neq 1.$$

Preoblikujemo prvu jednadžbu.

$$\frac{x^2}{y}+\frac{y^2}{x}=\frac{91}{12} \Rightarrow \frac{x^2}{y}+\frac{y^2}{x}=\frac{91}{12} \; /\cdot 12\cdot x\cdot y \Rightarrow 12\cdot x^3+12\cdot y^3=91\cdot x\cdot y \Rightarrow$$

$$\Rightarrow\ 12\cdot\left(x^3+y^3\right)=91\cdot x\cdot y\ \Rightarrow\ 12\cdot(x+y)\cdot\left(x^2-x\cdot y+y^2\right)=91\cdot x\cdot y\ \Rightarrow\ \left[\,x+y=7\,\right]\ \Rightarrow$$

$$\Rightarrow\ 12\cdot 7\cdot\left(x^2-x\cdot y+y^2\right)=91\cdot x\cdot y\ \Rightarrow\ 12\cdot 7\cdot\left(x^2-x\cdot y+y^2\right)=91\cdot x\cdot y\ /:7\ \Rightarrow$$

$$\Rightarrow\ 12\cdot x^2-12\cdot x\cdot y+12\cdot y^2=13\cdot x\cdot y\ \Rightarrow\ 12\cdot x^2-12\cdot x\cdot y+12\cdot y^2-13\cdot x\cdot y=0\ \Rightarrow$$

$$\Rightarrow\ 12\cdot x^2-25\cdot x\cdot y+12\cdot y^2=0.$$

Riješimo sustav.

$$\left.\begin{array}{l} x+y=7 \\ 12\cdot x^2-25\cdot x\cdot y+12\cdot y^2=0 \end{array}\right\}\ \Rightarrow\ \left.\begin{array}{l} y=7-x \\ 12\cdot x^2-25\cdot x\cdot y+12\cdot y^2=0 \end{array}\right\}\ \Rightarrow\ \left[\begin{array}{l}\text{metoda}\\ \text{zamjene}\end{array}\right]\ \Rightarrow$$

$$\Rightarrow\ 12\cdot x^2-25\cdot x\cdot(7-x)+12\cdot(7-x)^2=0\ \Rightarrow$$

$$\Rightarrow\ 12\cdot x^2-175\cdot x+25\cdot x^2+12\cdot\left(49-14\cdot x+x^2\right)=0\ \Rightarrow$$

$$\Rightarrow\ 12\cdot x^2-175\cdot x+25\cdot x^2+588-168\cdot x+12\cdot x^2=0\ \Rightarrow\ 49\cdot x^2-343\cdot x+588=0\ \Rightarrow$$

$$\Rightarrow\ 49\cdot x^2-343\cdot x+588=0\ /:49\ \Rightarrow\ x^2-7\cdot x+12=0\ \Rightarrow\ \left.\begin{array}{l} x^2-7\cdot x+12=0 \\ a=1\ ,\ b=-7\ ,\ c=12 \end{array}\right\}\ \Rightarrow$$

$$\Rightarrow\ \left.\begin{array}{l} a=1\ ,\ b=-7\ ,\ c=12 \\ x_{1,2}=\dfrac{-b\pm\sqrt{b^2-4\cdot a\cdot c}}{2\cdot a} \end{array}\right\}\ \Rightarrow\ x_{1,2}=\frac{-(-7)\pm\sqrt{(-7)^2-4\cdot 1\cdot 12}}{2\cdot 1}\ \Rightarrow\ x_{1,2}=\frac{7\pm\sqrt{49-48}}{2}\ \Rightarrow$$

$$\Rightarrow\ x_{1,2}=\frac{7\pm\sqrt{1}}{2}\ \Rightarrow\ x_{1,2}=\frac{7\pm 1}{2}\ \Rightarrow\ \left.\begin{array}{l} x_1=\dfrac{7+1}{2} \\ x_2=\dfrac{7-1}{2} \end{array}\right\}\ \Rightarrow\ \left.\begin{array}{l} x_1=\dfrac{8}{2} \\ x_2=\dfrac{6}{2} \end{array}\right\}\ \Rightarrow\ \left.\begin{array}{l} x_1=\dfrac{8}{2} \\ x_2=\dfrac{6}{2} \end{array}\right\}\ \Rightarrow\ \left.\begin{array}{l} x_1=4 \\ x_2=3 \end{array}\right\}.$$

Računamo y_1 i y_2.

- $$\left.\begin{array}{l} x=4 \\ x+y=7 \end{array}\right\}\ \Rightarrow\ 4+y=7\ \Rightarrow\ y=7-4\ \Rightarrow\ y_1=3.$$

Rješenje je uređen par

$$\left(x_1,\ y_1\right)=(4,\ 3).$$

- $$\left.\begin{array}{l} x=3 \\ x+y=7 \end{array}\right\}\ \Rightarrow\ 3+y=7\ \Rightarrow\ y=7-3\ \Rightarrow\ y_2=4.$$

Rješenje je uređen par

$$\left(x_2,\ y_2\right)=(3,\ 4).$$

Vježba 154

Odmor!

Rezultat: …

Zadatak 155 (Filip, gimnazija)

Riješite sustav: $\begin{cases} \dfrac{1}{x}+\dfrac{1}{y}=2 \\ \dfrac{1}{x^2}+\dfrac{1}{y^2}=2 \end{cases}.$

Rješenje 155

Ponovimo!

$$(a+b)^2=a^2+2\cdot a\cdot b+b^2 \quad , \quad \left(\frac{a}{b}\right)^n=\frac{a^n}{b^n} \quad , \quad \frac{a}{b}=1 \Rightarrow a=b \quad , \quad a^1=a.$$

$$a^n\cdot a^m=a^{n+m} \quad , \quad (a-b)^2=a^2-2\cdot a\cdot b+b^2 \quad , \quad a^n=0 \Rightarrow a=0 \quad , \quad a^n:a^m=a^{n-m}.$$

1.inačica

Preoblikujemo prvu jednadžbu.

$$\frac{1}{x}+\frac{1}{y}=2 \Rightarrow \frac{1}{x}+\frac{1}{y}=2 \ /^2 \Rightarrow \left(\frac{1}{x}+\frac{1}{y}\right)^2=2^2 \Rightarrow \frac{1}{x^2}+\frac{2}{x\cdot y}+\frac{1}{y^2}=4 \Rightarrow$$

$$\Rightarrow \left[\frac{1}{x^2}+\frac{1}{y^2}=2\right] \Rightarrow 2+\frac{2}{x\cdot y}=4 \Rightarrow \frac{2}{x\cdot y}=4-2 \Rightarrow \frac{2}{x\cdot y}=2 \Rightarrow \frac{2}{x\cdot y}=2 \ /:2 \Rightarrow$$

$$\Rightarrow \frac{1}{x\cdot y}=1 \Rightarrow x\cdot y=1 \Rightarrow x\cdot y=1 \ /\cdot\frac{1}{x} \Rightarrow y=\frac{1}{x}.$$

Promatramo sustav

$$\left.\begin{array}{l} y=\dfrac{1}{x} \\ \dfrac{1}{x}+\dfrac{1}{y}=2 \end{array}\right\} \Rightarrow y+\frac{1}{y}=2 \Rightarrow y+\frac{1}{y}=2 \ /\cdot y \Rightarrow y^2+1=2\cdot y \Rightarrow y^2-2\cdot y+1=0 \Rightarrow$$

$$\Rightarrow (y-1)^2=0 \Rightarrow y-1=0 \Rightarrow y=1.$$

Računamo x.

$$\left.\begin{array}{l} y=1 \\ y=\dfrac{1}{x} \end{array}\right\} \Rightarrow 1=\frac{1}{x} \Rightarrow x=1.$$

Rješenje je uređeni par

$$(x,\ y)=(1,\ 1).$$

2.inačica

$$\left.\begin{array}{l} \dfrac{1}{x}+\dfrac{1}{y}=2 \\ \dfrac{1}{x^2}+\dfrac{1}{y^2}=2 \end{array}\right\} \Rightarrow \left.\begin{array}{l} \dfrac{1}{y}=2-\dfrac{1}{x} \\ \dfrac{1}{x^2}+\dfrac{1}{y^2}=2 \end{array}\right\} \Rightarrow \left[\begin{array}{c} \text{metoda} \\ \text{zamjene} \end{array}\right] \Rightarrow \frac{1}{x^2}+\left(2-\frac{1}{x}\right)^2=2 \Rightarrow$$

$$\Rightarrow \frac{1}{x^2}+4-\frac{4}{x}+\frac{1}{x^2}=2 \Rightarrow \frac{1}{x^2}+4-\frac{4}{x}+\frac{1}{x^2}=2 \ /\cdot x^2 \Rightarrow 1+4\cdot x^2-4\cdot x+1=2\cdot x^2 \Rightarrow$$

$$\Rightarrow 1+4\cdot x^2-4\cdot x+1-2\cdot x^2=0 \Rightarrow 2\cdot x^2-4\cdot x+2=0 \Rightarrow 2\cdot x^2-4\cdot x+2=0 \ /:2 \Rightarrow$$

$$\Rightarrow \; x^2 - 2 \cdot x + 1 = 0 \;\Rightarrow\; (x-1)^2 = 0 \;\Rightarrow\; x - 1 = 0 \;\Rightarrow\; x = 1.$$

Računamo y.

$$\left.\begin{array}{l} x = 1 \\ y = 2 - \dfrac{1}{x} \end{array}\right\} \Rightarrow \; y = 2 - \frac{1}{1} \;\Rightarrow\; y = 2 - 1 \;\Rightarrow\; y = 1.$$

Rješenje je uređeni par

$$(x,\ y) = (1,\ 1).$$

3.inačica

Preoblikujemo drugu jednadžbu.

$$\frac{1}{x^2} + \frac{1}{y^2} = 2 \;\Rightarrow\; \frac{1}{x^2} + \frac{2}{x \cdot y} + \frac{1}{y^2} - \frac{2}{x \cdot y} = 2 \;\Rightarrow\; \left(\frac{1}{x} + \frac{1}{y}\right)^2 - \frac{2}{x \cdot y} = 2 \;\Rightarrow\; \left[\frac{1}{x} + \frac{1}{y} = 2\right] \Rightarrow$$

$$\Rightarrow\; 2^2 - \frac{2}{x \cdot y} = 2 \;\Rightarrow\; 4 - \frac{2}{x \cdot y} = 2 \;\Rightarrow\; -\frac{2}{x \cdot y} = 2 - 4 \;\Rightarrow\; -\frac{2}{x \cdot y} = -2 \;\Rightarrow$$

$$\Rightarrow\; -\frac{2}{x \cdot y} = -2 \; / \cdot \left(-\frac{1}{2}\right) \;\Rightarrow\; \frac{1}{x \cdot y} = 1 \;\Rightarrow\; x \cdot y = 1 \;\Rightarrow\; x \cdot y = 1 \; / \cdot \frac{1}{x} \;\Rightarrow\; y = \frac{1}{x}.$$

Promatramo sustav

$$\left.\begin{array}{l} y = \dfrac{1}{x} \\ \dfrac{1}{x} + \dfrac{1}{y} = 2 \end{array}\right\} \Rightarrow\; y + \frac{1}{y} = 2 \;\Rightarrow\; y + \frac{1}{y} = 2 \; / \cdot y \;\Rightarrow\; y^2 + 1 = 2 \cdot y \;\Rightarrow\; y^2 - 2 \cdot y + 1 = 0 \;\Rightarrow$$

$$\Rightarrow\; (y-1)^2 = 0 \;\Rightarrow\; y - 1 = 0 \;\Rightarrow\; y = 1.$$

Računamo x.

$$\left.\begin{array}{l} y = 1 \\ y = \dfrac{1}{x} \end{array}\right\} \Rightarrow\; 1 = \frac{1}{x} \;\Rightarrow\; x = 1.$$

Rješenje je uređeni par

$$(x,\ y) = (1,\ 1).$$

Vježba 155

Odmor!

Rezultat: …

Zadatak 156 (Dado, tehnička škola)

Riješite sustav: $\left\{\begin{array}{l} x - y = -1 \\ x^2 - y^2 = -21 \end{array}\right.$.

Rješenje 156

Ponovimo!

$$(a-b)^2 = a^2 - 2 \cdot a \cdot b + b^2 \quad , \quad a^2 - b^2 = (a-b) \cdot (a+b).$$

1.inačica

$$\left.\begin{array}{l} x - y = -1 \\ x^2 - y^2 = -21 \end{array}\right\} \Rightarrow \left.\begin{array}{l} x = y - 1 \\ x^2 - y^2 = -21 \end{array}\right\} \Rightarrow \left[\begin{array}{l} \text{metoda} \\ \text{zamjene} \end{array}\right] \Rightarrow\; (y-1)^2 - y^2 = -21 \;\Rightarrow$$

$$\Rightarrow\ y^2-2\cdot y+1-y^2=-21\ \Rightarrow\ y^2-2\cdot y+1-y^2=-21\ \Rightarrow\ -2\cdot y+1=-21\ \Rightarrow$$
$$\Rightarrow\ -2\cdot y=-21-1\ \Rightarrow\ -2\cdot y=-22\ \Rightarrow\ -2\cdot y=-22\ /:(-2)\ \Rightarrow\ y=11.$$

Računamo x.

$$\left.\begin{array}{l} y=11 \\ x-y=-1 \end{array}\right\}\ \Rightarrow\ x-11=-1\ \Rightarrow\ x=-1+11\ \Rightarrow\ x=10.$$

Rješenje je uređeni par

$$(x,\ y)=(10,\ 11).$$

2.inačica

$$\left.\begin{array}{l} x-y=-1 \\ x^2-y^2=-21 \end{array}\right\}\ \Rightarrow\ \left.\begin{array}{l} x=y-1 \\ (x-y)\cdot(x+y)=-21 \end{array}\right\}\ \Rightarrow\ -1\cdot(x+y)=-21\ \Rightarrow\ -1\cdot(x+y)=-21\ /\cdot(-1)\ \Rightarrow$$
$$\Rightarrow\ x+y=21.$$

Promatramo sustav jednadžba.

$$\left.\begin{array}{l} x-y=-1 \\ x+y=21 \end{array}\right\}\ \Rightarrow\ \left[\begin{array}{l} \text{metoda suprotnih} \\ \text{koeficijenata} \end{array}\right]\ \Rightarrow\ 2\cdot x=20\ \Rightarrow\ 2\cdot x=20\ /:2\ \Rightarrow\ x=10.$$

Računamo y.

$$\left.\begin{array}{l} x=10 \\ x+y=21 \end{array}\right\}\ \Rightarrow\ 10+y=21\ \Rightarrow\ y=21-10\ \Rightarrow\ y=11.$$

Rješenje je uređeni par

$$(x,\ y)=(10,\ 11).$$

Vježba 156

Odmor!

Rezultat: …

Zadatak 157 (Miro, maturant)

Razlika kvadrata dvaju prirodnih brojeva je 100. Koliki je umnožak tih brojeva?

A. 624 *B.* 584 *C.* 496 *D.* 434

Rješenje 157

Ponovimo!

$$a^2-b^2=(a-b)\cdot(a+b).$$

Skup prirodnih brojeva označavamo slovom N, a zapisujemo

$$N=\{1,\ 2,\ 3,\ 4,\ 5,\ \ldots,\ n-1,\ n,\ n+1,\ \ldots\}.$$

Neka su x i y prirodni brojevi za koje vrijedi uvjet u zadatku.

$$x^2-y^2=100\ \Rightarrow\ (x-y)\cdot(x+y)=100.$$

Budući da je riječ o umnošku prirodnih brojeva, dobit ćemo sljedeće sustave:

$$(x-y)\cdot(x+y)=100\ \Rightarrow\ \left.\begin{array}{l} x-y=1\ ,\ x+y=100 \\ x-y=2\ ,\ x+y=50 \\ x-y=4\ ,\ x+y=25 \\ x-y=5\ ,\ x+y=20 \end{array}\right\}.$$

Riješimo svaki sustav.

- $\left.\begin{array}{l} x-y=1 \\ x+y=100 \end{array}\right\} \Rightarrow \left[\begin{array}{l} \text{metoda suprotnih} \\ \text{koeficijenata} \end{array}\right] \Rightarrow 2\cdot x=101 \Rightarrow 2\cdot x=101 \;/:2 \Rightarrow x=\frac{101}{2}.$

Sustav nema prirodna rješenja jer x nije prirodan broj.

- $\left.\begin{array}{l} x-y=2 \\ x+y=50 \end{array}\right\} \Rightarrow \left[\begin{array}{l} \text{metoda suprotnih} \\ \text{koeficijenata} \end{array}\right] \Rightarrow 2\cdot x=52 \Rightarrow 2\cdot x=52 \;/:2 \Rightarrow x=26.$

Računamo y.

$$\left.\begin{array}{l} x=26 \\ x+y=50 \end{array}\right\} \Rightarrow 26+y=50 \Rightarrow y=50-26 \Rightarrow y=24.$$

Sustav ima prirodna rješenja. Umnožak brojeva je 624.

$$26\cdot 24=624.$$

- $\left.\begin{array}{l} x-y=4 \\ x+y=25 \end{array}\right\} \Rightarrow \left[\begin{array}{l} \text{metoda suprotnih} \\ \text{koeficijenata} \end{array}\right] \Rightarrow 2\cdot x=29 \Rightarrow 2\cdot x=29 \;/:2 \Rightarrow x=\frac{29}{2}.$

Sustav nema prirodna rješenja jer x nije prirodan broj.

- $\left.\begin{array}{l} x-y=5 \\ x+y=20 \end{array}\right\} \Rightarrow \left[\begin{array}{l} \text{metoda suprotnih} \\ \text{koeficijenata} \end{array}\right] \Rightarrow 2\cdot x=25 \Rightarrow 2\cdot x=25 \;/:2 \Rightarrow x=\frac{25}{2}.$

Sustav nema prirodna rješenja jer x nije prirodan broj.

Odgovor je pod A.

Vježba 157

Razlika kvadrata dvaju prirodnih brojeva je 60. Koliki je umnožak tih brojeva?

A. 124 *B.* 224 *C.* 284 *D.* 306

Rezultat: B.

Zadatak 158 (Lidija, srednja škola)

Riješite sustav: $\left\{\begin{array}{l} x+y-z=7 \\ \frac{x}{2}+\frac{2\cdot y}{3}-\frac{z}{4}=5 \\ \frac{x}{2}-\frac{2\cdot y}{3}+\frac{z}{4}=3 \end{array}\right..$

Rješenje 158

Ponovimo!

$$\frac{a}{n}+\frac{b}{n}=\frac{a+b}{n} \quad , \quad \left.\begin{array}{l} a=b \\ c=d \end{array}\right\} \Rightarrow a+c=b+d.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \;,\; n\neq 0 \;,\; n\neq 1.$$

Zbrojimo drugu i treću jednadžbu.

$$\left.\begin{array}{l} \frac{x}{2}+\frac{2\cdot y}{3}-\frac{z}{4}=5 \\ \frac{x}{2}-\frac{2\cdot y}{3}+\frac{z}{4}=3 \end{array}\right\} \Rightarrow \left[\begin{array}{l} \text{zbrojimo} \\ \text{jednadžbe} \end{array}\right] \Rightarrow \frac{x}{2}+\frac{2\cdot y}{3}-\frac{z}{4}+\frac{x}{2}-\frac{2\cdot y}{3}+\frac{z}{4}=5+3 \Rightarrow$$

$$\Rightarrow \frac{x}{2}+\frac{2\cdot y}{3}-\frac{z}{4}+\frac{x}{2}-\frac{2\cdot y}{3}+\frac{z}{4}=8 \Rightarrow \frac{x}{2}+\frac{x}{2}=8 \Rightarrow x=8.$$

Sada promatramo sustav

$$\left.\begin{aligned} x+y-z&=7 \\ \frac{x}{2}+\frac{2\cdot y}{3}-\frac{z}{4}&=5 \end{aligned}\right\} \Rightarrow [x=8] \Rightarrow \left.\begin{aligned} 8+y-z&=7 \\ \frac{8}{2}+\frac{2\cdot y}{3}-\frac{z}{4}&=5 \end{aligned}\right\} \Rightarrow \left.\begin{aligned} y-z&=7-8 \\ 4+\frac{2\cdot y}{3}-\frac{z}{4}&=5 \end{aligned}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{aligned} y-z&=-1 \\ \frac{2\cdot y}{3}-\frac{z}{4}&=5-4 \end{aligned}\right\} \Rightarrow \left.\begin{aligned} y-z&=-1 \\ \frac{2\cdot y}{3}-\frac{z}{4}&=1 \end{aligned}\right\} \Rightarrow \left.\begin{aligned} y-z&=-1 \\ \frac{2\cdot y}{3}-\frac{z}{4}&=1 \ /\cdot(-12) \end{aligned}\right\} \Rightarrow \left.\begin{aligned} y-z&=-1 \\ -8\cdot y+3\cdot z&=-12 \end{aligned}\right\} \Rightarrow$$

$$\Rightarrow \left[\begin{aligned} &\text{metoda suprotnih} \\ &\text{koeficijenata} \end{aligned}\right] \Rightarrow \left.\begin{aligned} y-z&=-1 \ /\cdot 3 \\ -8\cdot y+3\cdot z&=-12 \end{aligned}\right\} \Rightarrow \left.\begin{aligned} 3\cdot y-3\cdot z&=-3 \\ -8\cdot y+3\cdot z&=-12 \end{aligned}\right\} \Rightarrow$$

$$\Rightarrow -5\cdot y=-15 \Rightarrow -5\cdot y=-15 \ /:(-5) \Rightarrow y=3.$$

Računamo z.

$$\left.\begin{aligned} y&=3 \\ y-z&=-1 \end{aligned}\right\} \Rightarrow 3-z=-1 \Rightarrow -z=-1-3 \Rightarrow -z=-4 \Rightarrow -z=-4 \ /\cdot(-1) \Rightarrow z=4.$$

Rješenje sustava glasi:

$$(x,\ y,\ z)=(8,\ 3,\ 4).$$

Vježba 158

Odmor!

Rezultat: …

Zadatak 159 (Lidija, srednja škola)

Riješite sustav: $$\begin{cases} x_1+x_2+x_3+x_4=0 \\ x_2+x_3+x_4+x_5=9 \\ x_3+x_4+x_5+x_1=8 \\ x_4+x_5+x_1+x_2=7 \\ x_5+x_1+x_2+x_3=6 \end{cases}.$$

Rješenje 159

Ponovimo!

$$n=\frac{n}{1} \quad , \quad \frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d} \quad , \quad \left.\begin{aligned} a&=b \\ c&=d \end{aligned}\right\} \Rightarrow a+c=b+d.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \ , \ n\neq 0 \ , \ n\neq 1.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$\left.\begin{aligned} x_1+x_2+x_3+x_4&=0\\ x_2+x_3+x_4+x_5&=9\\ x_3+x_4+x_5+x_1&=8\\ x_4+x_5+x_1+x_2&=7\\ x_5+x_1+x_2+x_3&=6 \end{aligned}\right\} \Rightarrow \left[\begin{matrix}\text{zbrojimo}\\ \text{jednadžbe}\end{matrix}\right] \Rightarrow$$

$$\Rightarrow x_1+x_2+x_3+x_4+x_2+x_3+x_4+x_5+x_3+x_4+x_5+x_1+x_4+x_5+x_1+x_2+x_5+x_1+x_2+x_3=$$
$$=30 \Rightarrow$$

$$\Rightarrow 4\cdot x_1+4\cdot x_2+4\cdot x_3+4\cdot x_4+4\cdot x_5=30 \Rightarrow 4\cdot\left(x_1+x_2+x_3+x_4+x_5\right)=30 \Rightarrow$$

$$\Rightarrow 4\cdot\left(x_1+x_2+x_3+x_4+x_5\right)=30 \ /:4 \Rightarrow x_1+x_2+x_3+x_4+x_5=\frac{30}{4} \Rightarrow$$

$$\Rightarrow x_1+x_2+x_3+x_4+x_5=\frac{30}{4} \Rightarrow x_1+x_2+x_3+x_4+x_5=\frac{15}{2}.$$

Računamo x_1.

$$\left.\begin{aligned} x_2+x_3+x_4+x_5&=9\\ x_1+x_2+x_3+x_4+x_5&=\frac{15}{2} \end{aligned}\right\} \Rightarrow \left.\begin{aligned} x_2+x_3+x_4+x_5&=9\\ x_1+\left(x_2+x_3+x_4+x_5\right)&=\frac{15}{2} \end{aligned}\right\} \Rightarrow x_1+9=\frac{15}{2} \Rightarrow$$

$$\Rightarrow x_1=\frac{15}{2}-9 \Rightarrow x_1=\frac{15}{2}-\frac{9}{1} \Rightarrow x_1=\frac{15-18}{2} \Rightarrow x_1=-\frac{3}{2}.$$

Računamo x_2.

$$\left.\begin{aligned} x_3+x_4+x_5+x_1&=8\\ x_1+x_2+x_3+x_4+x_5&=\frac{15}{2} \end{aligned}\right\} \Rightarrow \left.\begin{aligned} x_3+x_4+x_5+x_1&=8\\ x_2+\left(x_3+x_4+x_5+x_1\right)&=\frac{15}{2} \end{aligned}\right\} \Rightarrow x_2+8=\frac{15}{2} \Rightarrow$$

$$\Rightarrow x_2=\frac{15}{2}-8 \Rightarrow x_2=\frac{15}{2}-\frac{8}{1} \Rightarrow x_2=\frac{15-16}{2} \Rightarrow x_2=-\frac{1}{2}.$$

Računamo x_3.

$$\left.\begin{aligned} x_4+x_5+x_1+x_2&=7\\ x_1+x_2+x_3+x_4+x_5&=\frac{15}{2} \end{aligned}\right\} \Rightarrow \left.\begin{aligned} x_4+x_5+x_1+x_2&=7\\ x_3+\left(x_4+x_5+x_1+x_2\right)&=\frac{15}{2} \end{aligned}\right\} \Rightarrow x_3+7=\frac{15}{2} \Rightarrow$$

$$\Rightarrow x_3=\frac{15}{2}-7 \Rightarrow x_3=\frac{15}{2}-\frac{7}{1} \Rightarrow x_3=\frac{15-14}{2} \Rightarrow x_3=\frac{1}{2}.$$

Računamo x_4.

$$\left.\begin{aligned} x_5+x_1+x_2+x_3&=6\\ x_1+x_2+x_3+x_4+x_5&=\frac{15}{2} \end{aligned}\right\} \Rightarrow \left.\begin{aligned} x_5+x_1+x_2+x_3&=6\\ x_4+\left(x_5+x_1+x_2+x_3\right)&=\frac{15}{2} \end{aligned}\right\} \Rightarrow x_4+6=\frac{15}{2} \Rightarrow$$

$$\Rightarrow x_4=\frac{15}{2}-6 \Rightarrow x_4=\frac{15}{2}-\frac{6}{1} \Rightarrow x_4=\frac{15-12}{2} \Rightarrow x_4=\frac{3}{2}.$$

Računamo x_5.

$$x_1+x_2+x_3+x_4+x_5=\frac{15}{2} \Rightarrow \begin{bmatrix} x_1=-\frac{3}{2} \\ x_2=-\frac{1}{2} \\ x_3=\frac{1}{2} \\ x_4=\frac{3}{2} \end{bmatrix} \Rightarrow -\frac{3}{2}-\frac{1}{2}+\frac{1}{2}+\frac{3}{2}+x_5=\frac{15}{2} \Rightarrow$$

$$\Rightarrow -\frac{3}{2}-\frac{1}{2}+\frac{1}{2}+\frac{3}{2}+x_5=\frac{15}{2} \Rightarrow x_5=\frac{15}{2}.$$

Rješenje sustava glasi:

$$\left(x_1, x_2, x_3, x_4, x_5\right)=\left(-\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{15}{2}\right).$$

Vježba 159

Riješite sustav: $\begin{cases} x_1+x_2+x_3+x_4=6 \\ x_2+x_3+x_4+x_5=10 \\ x_3+x_4+x_5+x_1=9 \\ x_4+x_5+x_1+x_2=8 \\ x_5+x_1+x_2+x_3=7 \end{cases}$.

Rezultat: $\left(x_1, x_2, x_3, x_4, x_5\right)=(0, 1, 2, 3, 4).$

Zadatak 160 (Matko5, gimnazija)

Riješite sustav: $\begin{cases} x^2-5\cdot x\cdot y+4\cdot y^2=0 \\ x^2+2\cdot x-3\cdot y-2=0 \end{cases}$.

Rješenje 160

Ponovimo!

$$\frac{a^n}{b^n}=\left(\frac{a}{b}\right)^n \quad , \quad (a\cdot b)^n=a^n\cdot b^n \quad , \quad \sqrt{a\cdot b}=\sqrt{a}\cdot\sqrt{b}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \ , \ n\neq 0 \ , \ n\neq 1.$$

Promotrimo prvu jednadžbu.

$$x^2-5\cdot x\cdot y+4\cdot y^2=0 \Rightarrow \left[\, y\neq 0 \,\right] \Rightarrow x^2-5\cdot x\cdot y+4\cdot y^2=0 \ / \cdot \frac{1}{y^2} \Rightarrow$$

$$\Rightarrow \frac{x^2}{y^2}-\frac{5\cdot x\cdot y}{y^2}+\frac{4\cdot y^2}{y^2}=0 \Rightarrow \left(\frac{x}{y}\right)^2-\frac{5\cdot x\cdot y}{y^2}+\frac{4\cdot y^2}{y^2}=0 \Rightarrow \left(\frac{x}{y}\right)^2-5\cdot\frac{x}{y}+4=0 \Rightarrow$$

$$\Rightarrow \begin{bmatrix} \text{zamjena} \\ \frac{x}{y}=t \end{bmatrix} \Rightarrow t^2-5\cdot t+4=0 \Rightarrow \begin{bmatrix} \text{bolje džepno} \\ \text{računalo} \end{bmatrix} \Rightarrow$$  $$\Rightarrow$$

$$\Rightarrow \left.\begin{matrix} t_1 = 1 \\ t_2 = 4 \end{matrix}\right\}.$$

Prvi slučaj

$$\left.\begin{matrix} \frac{x}{y} = t \\ x^2 + 2 \cdot x - 3 \cdot y - 2 = 0 \end{matrix}\right\} \Rightarrow [\, t = 1 \,] \Rightarrow \left.\begin{matrix} \frac{x}{y} = 1 \\ x^2 + 2 \cdot x - 3 \cdot y - 2 = 0 \end{matrix}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{matrix} x = y \\ x^2 + 2 \cdot x - 3 \cdot y - 2 = 0 \end{matrix}\right\} \Rightarrow x^2 + 2 \cdot x - 3 \cdot x - 2 = 0 \Rightarrow x^2 - x - 2 = 0 \Rightarrow$$

$\Rightarrow$  $\Rightarrow \left.\begin{matrix} x_1 = 2 \\ x_2 = -1 \end{matrix}\right\}.$

Računamo y_1 i y_2.

- $\left.\begin{matrix} x = 2 \\ x = y \end{matrix}\right\} \Rightarrow y_1 = 2.$
- $\left.\begin{matrix} x = -1 \\ x = y \end{matrix}\right\} \Rightarrow y_2 = -1.$

Rješenja su:

$$(x_1, y_1) = (2, 2) \quad , \quad (x_2, y_2) = (-1, -1).$$

Drugi slučaj

$$\left.\begin{matrix} \frac{x}{y} = t \\ x^2 + 2 \cdot x - 3 \cdot y - 2 = 0 \end{matrix}\right\} \Rightarrow [\, t = 4 \,] \Rightarrow \left.\begin{matrix} \frac{x}{y} = 4 \\ x^2 + 2 \cdot x - 3 \cdot y - 2 = 0 \end{matrix}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{matrix} \frac{x}{y} = 4 \;/\cdot y \\ x^2 + 2 \cdot x - 3 \cdot y - 2 = 0 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x = 4 \cdot y \\ x^2 + 2 \cdot x - 3 \cdot y - 2 = 0 \end{matrix}\right\} \Rightarrow (4 \cdot y)^2 + 2 \cdot 4 \cdot y - 3 \cdot y - 2 = 0 \Rightarrow$$

$\Rightarrow 16 \cdot y^2 + 8 \cdot y - 3 \cdot y - 2 = 0 \Rightarrow 16 \cdot y^2 + 5 \cdot y - 2 = 0 \Rightarrow$  $\Rightarrow$

$$\Rightarrow \left.\begin{matrix} y_3 = \frac{-5 + 3 \cdot \sqrt{17}}{32} \\ y_4 = \frac{-5 - 3 \cdot \sqrt{17}}{32} \end{matrix}\right\}.$$

Računamo x_3 i x_4.

- $\left.\begin{matrix} y = \frac{-5 + 3 \cdot \sqrt{17}}{32} \\ x = 4 \cdot y \end{matrix}\right\} \Rightarrow x_3 = 4 \cdot \frac{-5 + 3 \cdot \sqrt{17}}{32} \Rightarrow x_3 = 4 \cdot \frac{-5 + 3 \cdot \sqrt{17}}{32} \Rightarrow x_3 = \frac{-5 + 3 \cdot \sqrt{17}}{8}.$
- $\left.\begin{matrix} y = \frac{-5 - 3 \cdot \sqrt{17}}{32} \\ x = 4 \cdot y \end{matrix}\right\} \Rightarrow x_4 = 4 \cdot \frac{-5 - 3 \cdot \sqrt{17}}{32} \Rightarrow x_4 = 4 \cdot \frac{-5 - 3 \cdot \sqrt{17}}{32} \Rightarrow x_4 = \frac{-5 - 3 \cdot \sqrt{17}}{8}.$

Rješenja su:

$$\left(x_3,\ y_3\right)=\left(\frac{-5+3\cdot\sqrt{17}}{8},\frac{-5+3\cdot\sqrt{17}}{32}\right)\quad,\quad\left(x_4,\ y_4\right)=\left(\frac{-5-3\cdot\sqrt{17}}{8},\frac{-5-3\cdot\sqrt{17}}{32}\right).$$

Vježba 160

Odmor!

Rezultat: …